INITIAL PROBLEM OF FLUIDIZATION

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Introduction

When a gas or a liquid in drop form flows through a granular layer or bed, the particles are acted upon by hydrodynamic forces which lead to a considerable change in the stressed state of the system by comparison with the static situation characterizing the same bed in the absence of a flow. For certain modes of flow, in particular, tensile normal stresses may appear in the bed, producing local breaks in the continuity of the bed and its actual breakdown as a loose, continuous medium. The particles then lose regular contacts with one another and pass into a suspended ("fluidized") state. Despite the large number of empirical investigations which have been made into various kinds of fluidized systems and the wide practical use of fluidization techniques in various branches of industry, the fundamental question as to the reasons and conditions underlying the passage of a particle bed into the fluidized state and as to the possible mechanisms of such a transition is still far from a satisfactory solution [1-6]. It is therefore essential to construct a clear physical model of initial fluidization so as to be able to explain the observed qualitatively differing pictures of the generation of the fluidized state from a unified point of view.

The problem of the transition of a granular bed into the fluidized state was considered in [7, 8] as that of the limiting equilibrium of loose material withstanding only compressive or slight (not exceeding the critical cohesive stress in modulus) tensile stresses. Actually the potential possibility of discontinuities appearing in the granular material does not necessarily signify its true fluidization, since the transition into the fluidized state also involves overcoming the forces of boundary friction, which was not taken into account in the earlier papers [7, 8]. The more realistic problem of the initial fluidization of a granular bed in the presence of slight friction at the walls was considered in [9], in which for the first time allowance was made for the possible existence of residual stresses due to the initial irreversible deformation of the granular bed, not vanishing when the weight of the bed was fully compensated by hydraulic forces. A model of the transition of a stationary granular bed into the fluidized state, based on the use of approximate relationships describing the stresses in the static state [10], was proposed in [11]; this facilitated a qualitative study of possible fluidization mechanisms both for ideally loose media and for media incorporating cohesion. In this paper we shall consider such mechanisms in more detail for the case of a granular bed fluidized by a rising flow of liquid in systems with vertical walls.

1. Model Representations

Let us consider a plane or axisymmetrical cylindrical apparatus filled with a granular bed. We regard the upper surface of the bed as plane and free from stress; H is the depth of the bed; R is the half-width of the plane or the radius of the cylindrical bed. The z' axis is directed vertically downward along the symmetry axis; the x' axis is normal to it; and the origin of coordinates lies on the free surface of the bed.

Let us introduce the dimensionless quantities

$$x = x'/R,\ z = z'/R,\ h = H/R. \tag{1.1}$$

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The equilibrium equations in the variables of Eq. (1.1) are written in the form

$$\partial\sigma_x/\partial x + \partial\tau/\partial z = 0,\ \partial\tau/\partial x + k\tau/x + \partial\sigma_z/\partial z = \Gamma = \gamma R, \tag{1.2}$$

where σ_x and σ_z are the horizontal and vertical normal stresses (we use the rule of signs normally employed in the mechanics of loose media, in which compressive stresses are regarded as positive); τ is the tangential stress; γ is the effective specific gravity of the loose medium; and the parameter k equals zero for the plane and unity for the axisymmetrical problem. Motion of the liquid inside the bed is regarded as absent.

In conformity with the representations employed in [9-11], in order to characterize the properties of the system we introduce the parameters $\varkappa$, $\varkappa_e$, α, and σ_c. The first of these is introduced on the basis of experiments relating to the one-dimensional homogeneous compression (for example, in the z direction) of a loose sample bounded by solid walls normal to the transverse directions; it represents the proportionality factor between the developing transverse and applied longitudinal stresses. The quantity $\varkappa$ depends on the microscopic characteristics of the particles of the medium, their type of packing, and also on the irreversible deformations accumulated by the system [9, 10]. If the structure of the granular bed is uniform, $\varkappa$ may be regarded as a constant characterizing the initial state of the bed and depending on the history of the latter (different dynamic actions, vibrations, etc.). In the presence of substantial irreversible deformations the value of $\varkappa$ may be considerably greater than its minimum value of $\varkappa_e$, which also depends on the type of packing and the properties of the particles, but corresponds to the reversible deformation of the medium in the linear-elasticity range (for example, that encountered in the well-known Hertz problem). Clearly, in a state of limiting equilibrium the parameters $\varkappa$ and $\varkappa_e$ may be expressed in terms of the corresponding angle of internal friction of the medium.

At the walls of the apparatus we regard the limiting condition for the tangential stress, i.e., in effect the condition for the forces of boundary friction, as being satisfied. This "saturation" of the boundary friction is a result of the natural settling of the medium, for example, under the influence of its own weight [10]. Thus, a condition corresponding to the Coulomb law should be satisfied at the wall:

$$|\tau| = \alpha\sigma_x, \tag{1.3}$$

where α is the tangent of the effective boundary-friction angle.

Finally the critical cohesive stress σ_c characterizes the effective cohesion of the medium and represents the modulus of the maximum tensile stress capable of being withstood by the medium without the appearance of discontinuities [9, 11]. For real fluidized systems the appearance of a nonzero stress σ_c is usually associated with the adhesion of the particles in the medium (especially substantial for fine particles [12]), the electrification of the particles or their electromagnetic interactions [13, 14], and the conglomeration of the particles by virtue of the formation of liquid menisci over their contact areas [15]. For simplicity, we neglect the possible adhesion of the particles to the walls or the bottom of the apparatus.

The use of Eq. (1.3) and the condition of proportionality between the normal stresses under conditions of uniform loading makes the system (1.2) statically determinate. For a bed with a free upper boundary z = 0 the approximate solution takes the form [10]

$$\begin{gathered}\sigma_x = [\Gamma/(1+k)\alpha][f(z) - x^2\varphi(z)],\ \sigma_z = [\Gamma/(1+k)\varkappa\alpha]f(z),\\ \tau = [\Gamma/(1+k)][f(z) - \varphi(z)]x.\end{gathered} \tag{1.4}$$

The form of the functions of z in (1.4) depends on the sign of the parameter

$$T = 1 - 2(1+k)\varkappa\alpha^2. \tag{1.5}$$

In order to be specific we shall confine attention to situations in which T > 0; we then have [10]

$$\begin{gathered}f(z) = 1 - \frac{(1+\nu)^2}{4\nu}e^{-\lambda_1 z} + \frac{(1-\nu)^2}{4\nu}e^{-\lambda_2 z},\ \nu^2 = T,\\ \varphi(z) = \frac{1-\nu^2}{4\nu}\left(e^{-\lambda_1 z} - e^{-\lambda_2 z}\right),\ \lambda_1 = \frac{1-\nu}{\alpha},\ \lambda_2 = \frac{1+\nu}{\alpha}.\end{gathered} \tag{1.6}$$

Equations (1.4)-(1.6) describe the static stressed state of the granular bed in the absence of a flow of liquid.

Now let the bed be penetrated by a uniform upward-directed stream at a rate of flow Q, the force of hydraulic interaction referred to the particles in unit volume being g(Q) — a large number of empirical relationships [3-6] have been devised for the calculation of g(Q). The flow of liquid leads to a reduction in the apparent specific gravity of the loose medium, which becomes equal to $\gamma - g(Q)$, and to a corresponding release of the medium from existing stresses. This stress release has the effect that the limiting condition for the stresses ceases to be satisfied even at the walls of the apparatus and the corresponding problem ceases to be statically determinate.

In order to determine the state of stress so developing we make use of a hypothesis according to which the additional deformation due to the hydraulic forces does not produce any change in the structure of the medium, i.e., it is completely reversible. This hypothesis is excellently supported by experiments based on measuring the electrical resistance of granular beds for various values of Q. The dependence of the resistance on Q is extremely weak [5] right up to the value of Q_0 corresponding to the onset of fluidization, i.e., it reflects a comparatively slight reduction in the areas of contact between contiguous particles as a result of elastic deformations, but no substantial change in the structure of the medium. Thus in order to relate the additional (vertical and horizontal) normal stresses due to the hydraulic forces we must use the value of $\varkappa_e$ corresponding to reversible elastic deformation. For the stressed state of a granular bed penetrated by a flow of liquid we thus obtain the following expressions:

$$\sigma_x = [(\varkappa\Gamma - \varkappa_e G)/(1+k)\varkappa\alpha]f(z) - [(\Gamma - G)/(1+k)\alpha]x^2\varphi(z),\ G = Rg(Q),$$
$$\sigma_z = [(\Gamma - G)/(1+k)\varkappa\alpha]f(z),\ \tau = [(\Gamma - G)/(1+k)][f(z) - \varphi(z)]x, \tag{1.7}$$

where as before f(z) and $\varphi(z)$ are defined in (1.6). Equations (1.7) are only meaningful in the case

$$Q \leqslant Q_0,\ \gamma = g(Q_0). \tag{1.8}$$

Although when $Q = Q_0$ the vertical normal and tangential stresses vanish identically, the bed retains a "residual" horizontal stress

$$\sigma_x^0 = \frac{(\varkappa - \varkappa_e)\,\Gamma}{(1+k)\,\varkappa\alpha}\,f(z), \tag{1.9}$$

depending, as already indicated, on the characteristics of the initial state of the granular bed, and only vanishing when the initial packing is sufficiently "loose," i.e., when there are no irreversible deformations and $\varkappa = \varkappa_e$. The residual stress creates thrust forces at the walls, these being retained even after complete neutralization of the weight of the particles by the hydraulic forces. The origin of the thrust forces is discussed in [5] in connection with their influence on the transition into the fluidized state.

When the rate of flow Q exceeds the value of Q_0 from (1.8), the effective force acting on the particles of the granular bed is directed upward. The action of this force promotes the development of breaks in the continuity of the bed, the disruption of stable contacts between the particles, and the transition of the particles into the fluidized state. However, the existing forces of boundary friction, substantial in the presence of the residual stress (1.9), prevent the decomposition of the bed and the actual suspension of the particles. Hence in order to analyze the conditions governing the transition into the fluidized state we must give explicit consideration to the stresses for $Q > Q_0$; it is convenient to do this separately for media with and without cohesion.

2. Fluidization of an Ideally Free-Running Granular Bed

Let us first assume that there is no cohesion so that $\sigma_c = 0$. At $Q = Q_0$ the granular bed decomposes essentially into individual noninteracting horizontal layers in which the residual stresses (1.9) continue to act. For a slight increase in Q, when the volume force $g(Q) - \gamma$ is directed upward but is still not too great, the equilibrium of the majority of such layers is ensured by the appearance of boundary-friction forces directed downward and restraining the material from motion in the direction of the flow. While these forces remain lower than the limiting values defined in (1.3) with σ_x from (1.9), the individual horizontal layers may be regarded as existing under identical conditions, and there are no physical reasons why interaction should occur between them or a change occur in the stresses (1.9). Thus in the lower part of the granular bed in which the residual stresses are fairly large the following equations are satisfied:

$$\sigma_x = \sigma_x^0,\ \partial\sigma_z/\partial z = 0 \quad (z_*(x) < z < h), \tag{2.1}$$

where $z_*(x)$ is a certain surface depending on Q as a parameter. By using (2.1) in the corresponding equilibrium equations we obtain the following for the tangential stresses in the lower part of the bed:

$$\tau = -[(G - \Gamma)/(1 + k)]x \quad (z > z_*(x)), \tag{2.2}$$

the negative sign in (2.2) corresponding to the force of boundary friction directed downward.

In the upper part of the granular bed, in which the residual stresses are weak, the boundary friction is insufficient to compensate the volume force and hold the material in a state of equilibrium, i.e., this part of the bed passes into the fluidized state. Thus the surface $z_*(x)$ defines the boundary between the stationary and fluidized parts of the bed, in which the quantity $z_w = z_*(1)$ has to be determined from the condition that the stress (2.2) at the wall should coincide with the limiting value calculated by substituting (1.9) into (1.3). This gives an equation for z_w:

$$f(z_w) = \frac{g(Q) - \gamma}{\gamma}\frac{\varkappa}{\varkappa - \varkappa_e}. \tag{2.3}$$

Equation (2.3) has a solution $0 \le z_w \le h$, if Q satisfies the inequality $Q_0 \le Q \le Q_h$, where Q_h is determined from

$$g(Q_h) = \gamma\left[1 + \frac{\varkappa - \varkappa_e}{\varkappa} f(h)\right]. \tag{2.4}$$

If $\varkappa = \varkappa_e$, i.e., residual stresses and strains are completely absent, then $Q_h = Q_0$, and all parts of the granular bed are fluidized at the same time when the rate of flow Q reaches the so-called "minimum fluidization velocity" Q_0 defined in (1.8). This is the first of the possible fluidization mechanisms, for which the dependence of the liquid pressure drop in the bed Δp on Q has an "ideal" character (curve OABCDE in Fig. 1). For $Q > Q_0$ the value of Δp is constant and equal to the weight of the loose bed per unit area of its cross section $\gamma H = \Gamma h$.

If, $\varkappa > \varkappa_e$, then fluidization takes place gradually, starting from the upper surface of the bed, and lies within the range (Q_0, Q_h) of the rate of flow Q. For $Q = Q_h$ and $Q = Q_0$ the fluidization "front" $z_*(x)$ coincides, respectively, with the lower and upper surfaces of the granular bed. The function $z_*(x)$ and the vertical normal stress in the lower part of the bed may be determined from the conditions that the tangential and normal stresses should vanish at this front:

$$n_x\sigma_x + n_z\tau = 0,\ n_x\tau + n_z\sigma_z = 0 \quad (z = z_*(x)), \tag{2.5}$$

where $\mathbf{n}$ is the unit vector of the normal. Allowing for (1.9) and (2.1)-(2.3), from the first equation of (2.5) and the relation $n_x/n_z = -dz_*/dx$, we have

$$\int_{z_w}^{z_*(x)} f(z)\,dz = \frac{\alpha}{2}\frac{g(Q) - \gamma}{\gamma}\frac{\varkappa}{\varkappa - \varkappa_e}(1 - x^2),$$

which enables us to find $z_*(x)$ for various Q. Using the expressions for f(z) and z_w from (1.6) and (2.3), we may write down simple asymptotic representations respectively holding true for small and large $z_*(x)$. In particular, at great depths, beneath the free surface, the curve $z_*(x)$ approaches a parabola.

From the second equation of (2.5) we obtain

$$\sigma_z = \frac{\alpha x}{\varkappa - \varkappa_e}\frac{g(Q) - \gamma}{\gamma}\frac{G - \Gamma}{1 + k}\frac{x^2}{f(z_*)} \qquad (z > z_*(x)).$$

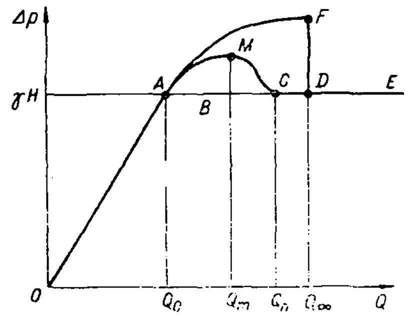

Fig. 1

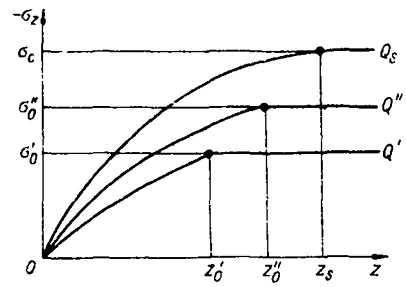

Fig. 2

Neglecting possible distortions of the flow of liquid close to the curved interface $z_*(x)$ between the fluidized and stationary parts of the bed, for the pressure drop in the bed we obtain

$$\Delta p = \Gamma z_0 + G(h - z_0) \geqslant \Gamma h, \; z_0(Q) = z_*(0).$$

Thus the pressure drop in the range (Q_0, Q_h) of the flow of liquid is greater than the weight of the loose layer. The fluidization curve $\Delta p(Q)$ has a maximum at a certain $Q = Q_m$, $Q_0 < Q_m < Q_h$ (curve OAMCDE in Fig. 1), the section MC (representing the fall in pressure) being the steeper, the greater the value of Q_h, i.e., the higher the bed. The maximum possible value Q_∞ of Q_h is reached in a bed of infinite height, and it follows from (1.6) and (2.4) that

$$g(Q_\infty) = \gamma(2 - \varkappa_e/\varkappa).$$

In this case the fluidization curve has the shape of curve OAFDE in Fig. 1. (For convenience we have not shown the differences between the initial sections OA of the different curves in Fig. 1, and all the curves are referred to the same ordinate Γh.)

In a bed of finite height the surface $z_*(x)$ moves continuously downward with increasing Q until its continuity is broken by touching the lower boundary $z = h$. For $z_0 \geq h$ (but $z_w < h$) the function $z_*(x)$ describes the surface of the stagnant zones formed at the walls within a comparatively narrow range of flow rates slightly smaller than Q_h. This gradual fluidization of the granular bed constitutes the second possible mechanism of initial fluidization.

Clearly, it is precisely this mechanism which is encountered most frequently in practice. The pressure maximum on the corresponding fluidization curve was earlier explained as being due to the loss of flow energy in accelerating the particles during the change in the structure of the layer [1] or in overcoming the cohesive forces [2, 4-6], the inhomogeneity of the initial packing of the bed and the hydraulic forces in the latter [3, 5], and so on. It follows from the foregoing analysis that these factors play no serious part in the creation of the maximum, the appearance of which is rather due to the presence of residual stresses. The latter possibility was intuitively admitted in [5].

The relative deviation of the real fluidization curve from the ideal shape increases sharply with increasing $\varkappa$ and h, so that the maximum pressure drop may greatly exceed the effective weight of the loose granular bed. These conclusions are excellently supported, for example, by the experiments on magnesium oxide particle beds described in [16].

For lower beds the deviations of the liquid flow from uniformity may become substantial on account of the curvature of the fluidization front and the difference between the hydraulic resistances of the particles in the stationary bed and in the fluidized state, and so may the random inhomogeneities in the packing of the bed. Analysis shows that both these circumstances promote the accelerated fluidization of the central regions of the bed by comparison with the peripheral regions, and may, under favorable conditions, lead to the formation of channels filled only with the suspended particles, and to the preferential flow of the liquid through such channels, with the formation of stagnant zones close to the walls.

3. Fluidization in the Presence of Cohesion

When the critical cohesive stress σ_c differs from zero, the condition for the breakdown of continuity in the granular bed and the appearance of breaks in the latter takes the form

$$\sigma_m \geqslant -\sigma_c, \tag{3.1}$$

where σ_m is the least of the principal normal stresses in the bed. We see from (3.1) and expressions (1.7) for the stresses that in this case fluidization can only begin when the flow of liquid reaches a certain value exceeding the minimum fluidization velocity.

As before, when $Q > Q_0$ a stressed state is created in the lower part of the granular bed, this state being governed by Eqs. (2.1) and (2.2). However, in the case under consideration the upper part of the bed is not necessarily in the fluidized state. For the suspension of the particles, in fact, it is essential to satisfy (3.1), which is physically unrealistic if the quantity $Q - Q_0$, although positive, is fairly small. Thus over a certain range of flow values starting from Q_0 a stressed state satisfying the limiting condition (1.3) at the wall will be created in the region adjacent to the upper surface of the bed. In this case the function $z_*(x)$ describes the shape of the interface between the region with the foregoing stressed state and the region corresponding to Eqs. (2.1) and (2.2), the position of z_w (the point of intersection of this surface with the wall) being given by Eq. (2.3) as before.

On the basis of the model of Sec. 1, we write the problem of determining the stressed state in the region above $z = z_*(x)$ in the form

$$\begin{gathered}\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau}{\partial z} = 0, \quad \frac{\partial \tau}{\partial x} + \frac{k\tau}{x} + \frac{\partial \sigma_z}{\partial z} = -G + \Gamma < 0,\\ \sigma_x = \frac{(\varkappa - \varkappa_e)\Gamma}{(1+k)\varkappa\alpha} f(z) + \sigma_x' \quad (z < z_*(x)),\\ \tau = \begin{cases}\alpha\sigma_x, & \sigma_x > 0\\ 0, & \sigma_x < 0\end{cases} \quad (x = 1), \quad \sigma_x' = \varkappa_e \sigma_z \quad (x = 0),\\ \tau = 0, \quad \sigma_z = 0 \quad (z = 0).\end{gathered} \tag{3.2}$$

Here we have taken account of Eq. (1.9) for the residual stresses (which simplifies our assumption as to the absence of cohesion with the walls) and also the fact that the hydraulic forces lead to a reversible elastic deformation of the free-running medium without disrupting its structure. It may be shown that the solution of Eq. (3.2) takes the form

$$\begin{gathered}\sigma_x' = \varkappa_e \Phi(z) + \frac{1}{2(1+k)} \frac{d^2\Phi}{dz^2} x^2,\\ \tau = [1/(1+k)](-G + \Gamma - d\Phi/dz)x, \quad \sigma_z = \Phi(z),\end{gathered} \tag{3.3}$$

in which the function $\Phi(z)$ is found from the solution of the simple linear problem which follows from (3.2) on substitution of (3.3) into the latter. In view of their cumbersome nature we shall not write out the explicit representations of this function or the stresses in the region $z < z_*(x)$, confining ourselves subsequently to a qualitative analysis of the solution.

The unknown shapes of the surface $z_*(x)$ and the vertical stress in the region $z > z_*(x)$ may simply be found from the conditions of continuity of the tangential and normal stresses on this surface, which in the present case replace conditions (2.5) and Eqs. (2.3) for z_w. In this way we obtain the following formal relationships, enabling us to complete the solution of the problem:

$$\begin{gathered}\frac{dz_*}{dx} = \frac{\tau^+ - \tau^-}{\sigma_x^+ - \sigma_x^-}, \quad z_*(1) = z_w,\\ \sigma_z^+ = \sigma_z^- + (\tau^+ - \tau^-)\frac{dz_*}{dx}.\end{gathered} \tag{3.4}$$

The right-hand sides in Eqs. (3.4) may be regarded as already known; the stresses at $z = z_* - 0$ and $z = z_* + 0$ are indicated by upper indices "–" and "+," respectively.

From the qualitative point of view, the stressed state so arising is illustrated in Fig. 2, which shows the characteristic dependences of the vertical stress at $x = 0$ on the coordinate z corresponding to different Q (the curves of Fig. 2 correspond to $Q_s > Q'' > Q' > Q_0$). In the region $z < z_0 = z_*(0)$ this stress falls monotonically to a certain limiting value $-\sigma_0$; in the region $z > z_0$ it is independent of z and equal to $-\sigma_0$. The quantity σ_0 increases monotonically with increasing flow Q, on which it depends as a parameter. It is clear that, on account of Eq. (3.1), physically realistic states only occur for such stressed states as satisfy the condition $\sigma_0 < \sigma_c$. When the flow reaches a certain critical value Q_s at which the tensile stress σ_0 becomes equal to σ_c, the continuity of the granular bed is infringed; close to the point $(0, z_s)$ a small horizontal break appears. The latter leads, first, to the partial relief of stresses along the borders of the break, i.e., above and below it, and, secondly, to the ordinary stress concentration at its outer contour, which promotes a further increase in the surface area of the break until it passes into the region close to the wall.

From the phenomenological point of view the foregoing process is extremely like the well-known picture of rapid crack propagation in a brittle material. As a result of this, a "piston" is separated from the granular bed, initiating motion in the direction of the suspending flow. The height of this piston is determined by the quantity $z_s = z_0$ corresponding to the value of Q_s, i.e., for determining z_s and Q_s we have a formal system consisting of the first equation of (3.4), with z_w from (2.3), and the equation

$$\Phi(z_s) = -\sigma_c.$$

After the separation of the first piston, on further increasing the flow further pistons are separated from the bed, their heights in general being different from one another.

This picture of piston formation represents the third possible mechanism of initial fluidization. This mechanism is actually realized if the bed is fairly high, i.e., $z_s < h$. In the opposite case, $z_s > h$, for a certain $Q_s' < Q_s$, at which the corresponding value of z_0 is comparable with h, the whole bed is detached from the lattice supporting it, i.e., fluidization is effected by a fourth possible mechanism.

Subsequently the particles falling from the lower surface of the bed separated from the lattice in this way either reform the stationary layer by gradually accumulating on the lattice or else pass directly into the suspended state, which corresponds to fluidization starting from the lower surface of the bed. The first process occurs if the initially broken adhesive bonds between the particles are restored sufficiently rapidly after the particles come into contact again. The second process occurs when the relaxation time of the bonds in question is great, i.e., the bonds are broken irreversibly.

We note that all the fluidization mechanisms discussed are in fact observed experimentally. The most complete phenomenological description of the mechanisms involved and a discussion of the factors leading to the practical realization of one version or another are to be found in [3].

In order to secure a further proof of the model we set up some special experiments (in conjunction with E. N. Prozorov) in which loose granular beds consisting initially of alternating horizontal layers of differently colored particles were fluidized. The boundary friction was varied by sticking emery paper of various grades to the walls of the cylindrical apparatus, while the degree of initial consolidation of the charge was varied by holding it for a long time under an additional load. The instantaneous state of the bed after gradually or abruptly increasing the rate of flow was recorded by rapidly excluding the gas and then roasting the deposited layer in a muffle furnace. The character of the transition into the fluidized state was judged from a study of the microsections of the sintered sample and from the fluidization curves recorded. The results of these experiments qualitatively supported all the main conclusions of the proposed model. Thus the height and extent of the peak on the fluidization curve increased with intensification of the initial packing of the bed and also with increasing height and boundary friction; the shape of the boundaries between the fluidized and stationary parts of the bed was similar to that calculated above; stagnant zones were formed at the walls when the central part of this boundary passed to the gas-distributing lattice, and so on.

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TWO-DIMENSIONAL EFFECTS WITH THE FLOW OF A REACTIVE LIQUID WITH PROPERTIES VARYING WITH THE DEPTH OF THE CONVERSION

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During the course of chemical conversions, the mechanical properties of a reacting liquid can vary. Thus, polymerization processes are usually accompanied by a considerable increase in the viscosity. This leads to the appearance of specific hydrodynamic effects. Some of these are considered in the present article using the example of the simplest two-dimensional problem.

§1. The article considers the steady-state laminar flow of a reactive Newtonian liquid in a tube. The viscosity μ and the density ρ of the liquid, during the course of chemical conversions, vary from the values $\mu = \mu_0$ and $\rho = \rho_0$ to the values $\mu = \mu_1$ and $\rho = \rho_1$ for total conversion.

We shall assume that the temperature of the liquid is constant and that the effect of diffusion can be neglected. In this case, at a given point, the depth of the conversion and the mechanical properties of the liquid are determined only by the time t at which the liquid reaches the given point. The dependences $\mu = \mu(t)$ and $\rho = \rho(t)$ are the same as in the case

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